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Harmonics and The Glass Bead Game

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This evening, I would like to talk about harmonics and the Glass Bead Game. "*The Glass Bead Game*" is - as is probably well known - the English title of a novel by Hermann Hesse: *Das Glasperlenspiel*. That does not mean, however, that I mean here to deliver a literary-historical lecture: the *Glass Bead Game* has - for me, at least, and certainly also for others - become a symbol for a particular way of thinking, for a particular way of searching for relations and connections. I have already delivered this lecture in a similar form on a previous occasion under the title: "*What Is Harmonics?*" However the title no longer pleased me, because it led you to expect that at the end of the lecture you would know what harmonics is, whereas, in fact, I don't even know myself. By which I mean: harmonics is a word that perhaps leads one to expect something that has to do with harmony in the aesthetic sense, perhaps also something ideological, or, on the other hand, with that which in music is called 'harmony', namely the study of the relationships between tones, how to harmonize a piece of music, how harmonic relationships are employed in music. Harmonics, naturally, does also have to do with that, i.e. the association is perfectly correct, a connection with music does exist. But, basically, it is about something far more comprehensive, and the way Hermann Hesse describes his *Glass Bead Game* represents very accurately what the term harmonics means for me. Let me read a few sentences from "*The Glass Bead Game*" where it could hardly be expressed more beautifully. Hesse says there:

“These rules, the sign language and grammar of the Game, constitute a kind of highly developed secret language drawing upon several sciences and arts, but especially mathematics and music (and/or musicology), and capable of expressing and establishing interrelationships between the content and conclusions of nearly all scholarly disciplines. The Glass Bead Game is thus a mode of playing with the total contents and values of our culture; it plays with them as, say, in the great age of the arts a painter might have played with the colors on his palette. All the insights, noble thoughts, and works of art that the human race has produced in its creative eras, all that subsequent periods of scholarly study have reduced to concepts and converted into intellectual property -- on all this immense body of intellectual

values the Glass Bead Game player plays like the organist on an organ. And this organ has attained an almost unimaginable perfection; its manuals and pedals range over the entire intellectual cosmos; its stops are almost beyond number. Theoretically this instrument is capable of reproducing in the Game the entire intellectual content of the universe."

That is, first of all, merely an intimation of what is meant. Already, however, one gets a sense of the ideas upon which it might be based and also that it has something to do with music. I would say that the main pillars upon which harmonics and the Glass Bead Game rest are firstly music, secondly mathematics, and thirdly philosophy. These ideas are of ancient origin, and I can also read a few more sentences from Hesse on the history of harmonics or of the Glass Bead Game:

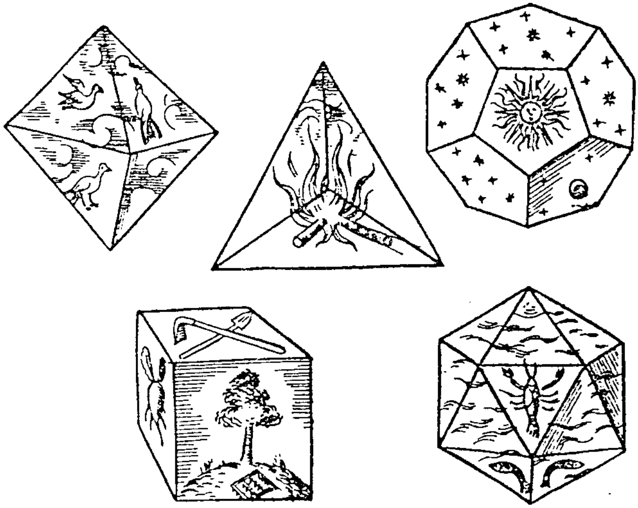
How far back the historian wishes to place the origins and antecedents of the Glass Bead Game is, ultimately, a matter of his personal choice. For like every great idea it has no real beginning; rather, it has always been, at least the idea of it. We find it foreshadowed, as a dim anticipation and hope, in a good many earlier ages. There are hints of it in Pythagoras, for example, and then among Hellenistic Gnostic circles in the late period of classical civilization. We find it equally among the ancient Chinese, then again at the several pinnacles of Arabic-Moorish culture; and the path of its prehistory leads on through Scholasticism and Humanism to the academies of mathematicians of the seventeenth and eighteenth centuries and on to the Romantic philosophies and the runes of Novalis's hallucinatory visions. This same eternal idea, which for us has been embodied in the Glass Bead Game, has underlain every movement of Mind toward the ideal goal of a universitas litterarum, every Platonic academy, every league of an intellectual elite, every rapprochement between the exact and the more liberal disciplines, every effort toward reconciliation between science and art or science and religion. Men like Abelard, Leibniz, and Hegel unquestionably were familiar with the dream of capturing the universe of the intellect in concentric systems, and pairing the living beauty of thought and art with the magical expressiveness of the exact sciences. In that age in which music and mathematics almost simultaneously attained classical heights, approaches and cross-fertilizations between the two disciplines occurred frequently. And two centuries earlier we find in Nicholas of Cues sentences of the same tenor, such as this: "The mind adapts itself to potentiality in order to measure everything in the mode of potentiality, and to absolute necessity in order to measure everything in the mode of unity and simplicity as God does, and to the necessity of nexus in order to measure everything with respect to its peculiar nature; finally, it adapts itself to determinate poten-

tiality in order to measure everything with respect to its existence. But furthermore the mind also measures symbolically, by comparison, as when it employs numerals and geometric figures and equates other things with them." Incidentally, this is not the only one of Nicholas's ideas that almost seems to suggest our Glass Bead Game, or corresponds to and springs from a similar branch of the imagination as the play of thought which occurs in the Game. Many similar echoes can be found in his writings. His pleasure in mathematics also, and his delight and skill in using constructions and axioms of Euclidean geometry as similes to clarify theological and philosophical concepts, likewise appear to be very close to the mentality of the Game. At times even his peculiar Latin (abounding in words of his own coinage, whose meaning, however, was perfectly plain to any Latin scholar) calls to mind the improvisatory agility of the Game's language.

We heard about Nicholas of Cues, of course, in the last lecture in this series and also about his connection with harmonics. Observations can be made about many individual figures in the history of ideas and their connection with harmonics. One person who has scarcely been mentioned in this citation is Plato; the only mention has been of the Platonic academies. Plato is regarded in harmonics as one of the most important precursors and someone in whose thought these ideas are very strongly developed.

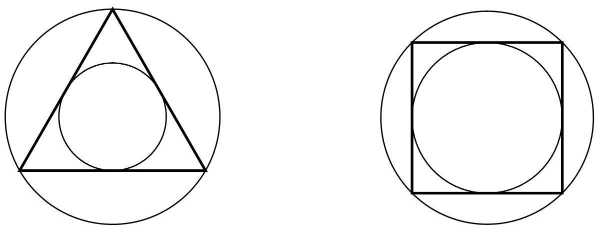
Another who has not been mentioned here is Johannes Kepler. Kepler played a very central role in harmonics. Before Kepler, you see, it was not all that common to look in a very concrete way into nature and inquire where in nature such harmonic laws were to be found. That is to say, observation proceeded far more from philosophic speculation: it was to a certain extent taken for granted that nature and the world - just like music - were constructed from measure and number. But Kepler was in fact the first to undertake the search for these laws in nature. Kepler, of course, is primarily known today as an astronomer, but he was far more an exponent of harmonics than an astronomer and had from his youth onwards let himself be guided in his study of the world by these ideas. And one of his first ideas was that the structure of the world, in this case, to be more precise, the structure of the planetary system might perhaps have something to do with the Platonic solids.

What are the Platonic solids? We see them here in this illustration: there are exactly five solids and there could be no more. They are defined by the fact that their faces are all regular polygons of the same type - i.e. triangles, squares or pentagons in each case with sides of the same length. No polyhedron can be constructed from the hexagon because it



extends in a plane, as we see in the honeycomb. Due to the structure of our space, then, only these five regular polyhedra can be formed. The simplest of the Platonic solids is the tetrahedron - a pyramid with four triangular faces. Equilateral triangles also make up the sides of the octahedron - a double pyramid - and the (twenty-sided) icosahedron. With the square, it is only possible to create one regular polyhedron, namely the cube. Similarly, the pentagon can only form the basis for one polyhedron and that is the dodecahedron, which is made up of twelve regular pentagons. Already in Plato's time, these polyhedra had been assigned to the elements, as Kepler has done in this illustration. And you can see very clearly here how - as is characteristic of this way of thinking - from the form, content can be derived. In other words, the 'thing in evidence' is not only observed from its quantitative side; one does not simply take note of the fact that we have four triangles here and eight there, but rather one sees that the whole has a shape, and that this shape has a content. One might consider now that this assignment is somewhat arbitrary and ask whether the bodies and elements might not be differently assigned to one another. What is important is that the old elements were understood as fundamental forces and that the question that has been asked here is "which fundamental force does the body in question express by its shape?" At two opposite poles, we have for example these two: the one is the most angular, the sharpest, namely the tetrahedron, which is formed by four triangles, and the other is the icosahedron, which is formed by twenty triangles and is, of all these solids, the one that comes closest to a sphere. Sharp and round - equated with dryness and fire on the one hand and with humidity and water on the other. This is plausible - imagine a dried fruit, which has very angular forms but becomes round again when immersed in water, or the roundness of a drop of water itself. This, then, would be one polarity, which is illustrated here: Fire and Water. Then, naturally, the cube, which rests so firmly on the earth, is assigned to Earth, as it is the body which, by its shape, exhibits the most weight. Contrasted with this is the octahedron, which it is in fact impossible to set down - you have to stand it on its end, so that it stands by virtue of its symmetry, as it is shown doing here. It can only hover in the air and is therefore the airiest body - so we get the polarity of Earth and Air. And what remains is this somewhat mysterious dodecahedron, formed by twelve pentagons, the quinta essentia or fifth element, assigned to the cosmos or Ether, which is represented by these stars. This body combines in itself the number twelve with the number five - i.e. the twelve signs of the Zodiac with the fifth element.

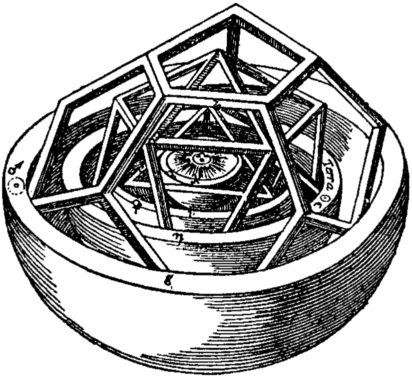
Kepler's idea now was that if you nest these bodies inside one another, in the manner suggested by their very nature, you obtain something approximating to the distances between the planets. I should perhaps quickly illustrate by means of a drawing the form this nesting takes.

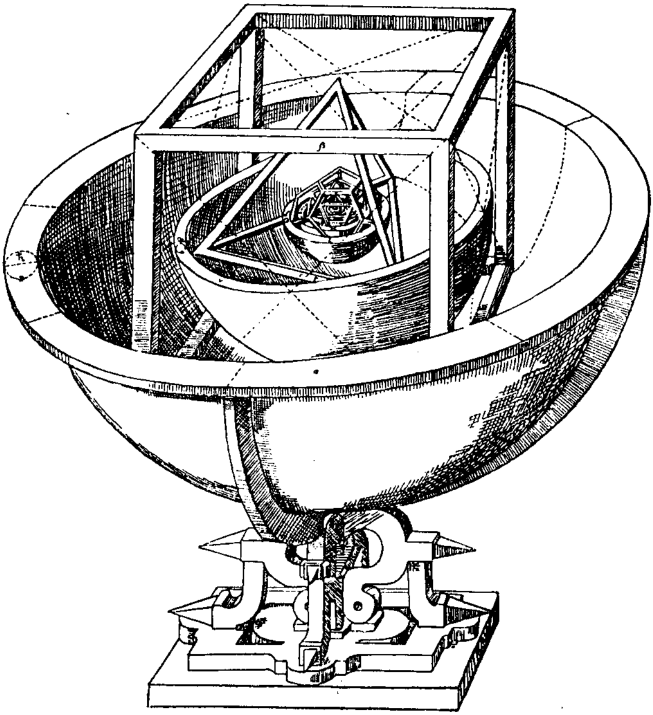


If, for example, I have a triangle, that triangle has an outer circle that touches all its vertices from the outside and an inner circle that touches all its sides from within. Much the same is true of these bodies: they have an inner sphere and an outer sphere. You can perhaps best visualize this if you picture one of these bodies inside an air balloon which then shrinks until it touches all the vertices of the object simultaneously from the outside. That would be the outer sphere. Now picture a second balloon inside the body which is inflated until it touches all its faces from the inside. That would be the inner sphere. And the size ratio of the inner and outer spheres of each of these bodies is something quite specific and characteristic of these bodies. To make this clearer, we should perhaps look again at our two-dimensional figures, in which we can see immediately that the difference between the inner circle and outer circle is greater in the case of the triangle than in that of the square - another consequence of the more pointed shape of the triangle. Now, it is possible to nest the various Platonic bodies inside one another in such a way that - looking from the centre outwards - the outer sphere of the first body is at the same time the inner sphere of the next, and so on. From this we obtain a given sequence of sphere radii.

In just this manner, Kepler inserted successively one Platonic body inside another until he obtained a model of the solar system - calling it *Mysterium Cosmographicum*. In this illustration from his book, which is also called *Mysterium Cosmographicum* ("The Cosmographic Mystery"), we see the succession of planetary spheres, previously presented directly as spherical shells. What remains to be noted is that this model is based on the assumption that the centre of the system is occupied by the Sun, with the Earth moving around it - a view by no means universally accepted in Kepler's day, even though Copernicus had some fifty years earlier established and published his finding that the movements of the planets could be described far more elegantly if the Sun was placed in the centre of the system. This idea, however, was even in Kepler's day still bitterly disputed - Kepler himself, however, took it for granted because for him it was axiomatic that the whole must necessarily be logical and beautiful - a view he had advocated since his youth. So he, too, placed the Sun in the centre of the system and arranged the Platonic bodies with their inner and outer spheres around it, deriving the path of the planets from these concentric shells. And, in fact, if the bodies are nested in the right order, the results do approximate relatively well to the paths of the planets known at that time. I say 'known at that time' advisedly, because, of course, other planets have been discovered since - the first of them, Uranus, 200 years later - whereas there can only be five Platonic bodies. Kepler even based his belief that there were six planets on the presence of just these five Platonic bodies interposed between them. The theory cannot, then, be entirely correct; nonetheless, it remains astounding how well it fits the facts - at least, as regards the inner planets.

For me here, though, it is not so much a question of whether or not Kepler's theory is correct or offers proof of the presence of a particular





order: I relate it simply to illustrate a particular way of thinking. Later Kepler adopted another approach. He had observed the angular speeds of the planets and looked for musical harmonies in them. The discoveries that he made in this way are what are taught in schools today as 'Kepler's Three Laws of Planetary Motion'. These state that each planet follows an elliptical course with the sun as one of the foci and that there is therefore one point in the orbit at which the planet is at its closest to, and another at which it is at its most distant from, the sun. When closer to the sun, the planet moves more quickly, and more slowly when further away, and these speeds, regarded from the sun outwards as the angle covered in a particular period of time, are the angular speeds. Kepler had studied these speeds and established that, in terms of their relationship with one another, they correspond very well to musical intervals. In his first approach, the geometry of the Mysterium Cosmographicum, the musical idea was not as pronounced, but with his second idea of world harmony - *Harmonices mundi* is the title of the book in which he outlines it - the entire solar system is represented as a music composition of the creator.

I had said that Kepler was one of the first to have looked into Nature in order to search for such harmonic laws. At the same time, however, another movement in research began with him and led eventually to what we know today as 'exact science'. From Galileo Galilei, Kepler's contemporary, derives the maxim: "*Measure what is measurable and make measurable what is not so*". Kepler must also have underlined this sentence, and this sentence inaugurated - not necessarily, but historically - a development in the sciences that led eventually to the purely quantitative observation of phenomena and to sight being lost of the aspect of their quality and design. That was, however, not yet the case in the immediate aftermath; we learn very little in school today about how the researchers of earlier times thought, because the 'positive' result is mostly seen as independent of the path by which it was arrived at. So, for example, we learn Kepler's three laws but not the fact that for Kepler these were merely a by-product of his search for musical harmony in the solar system; and hardly anyone is aware that Isaac Newton wrote more about theology and philosophy than about physics. Intellectuals such as Goethe or Novalis are not regarded as scientists but as poets or, at best, philosophers, simply because the results of their investigations cannot be slotted into the prevailing scientific world view without calling it into question. And yet Goethe actually regarded his work on colors and morphology as more important than his poetry.

There was certainly not as pronounced a separation between natural science and philosophy as there is today; researchers were to a greater or lesser extent polymaths. As a result of this separation, if we wish to follow the traces of harmonics or the Glass Bead Game, we must search for them from this period onwards more in philosophy than the natural sciences. One of these philosophers, who is also regarded as the last of the polymaths, was Leibniz, who was born towards the end of the Thirty Years War, sixteen years after Kepler's death. From Leibniz comes the statement that "*Music is the hidden mathematical endeavor of a soul unconscious it is calculating*". That sounds somewhat prosaic to our ears; those with a rather more emotional relationship with music who are told that music is in fact the subconscious calculation of the soul are unlikely to concur at all readily. What is meant, however, is that music and mathematics have roots in the same soil and that mathematics for this reason offers the same aesthetic and even religious edification as music - which, of course, is not something any true mathematician needs to be told.

I would like to read a short passage from Schopenhauer's "*The World as Will and Representation*" in which he says: "*In the whole of this discussion on music I have been trying to make it clear that music expresses in an exceedingly universal language, in a homogeneous material, that is, in mere tones, and with the greatest distinctness and truth, the inner being, the in-itself, of the world (...) Further, according to my view and contention, philosophy is nothing but a complete and accurate repetition and expression of the inner nature of the world in very general concepts (...) Thus whoever has followed me and has entered into my way of thinking will not find it so very paradoxical when I say that, supposing we succeeded in giving a perfectly accurate and complete explanation of music which goes into detail, and thus a detailed repetition in concepts of what it expresses, this would also be at once a sufficient repetition and explanation of the world in concepts, (...) and hence the true philosophy. Consequently, we can parody in the following way the above-mentioned saying of Leibniz, in the sense of our higher view of music, for it is quite correct from a lower point of view: 'Music is an unconscious exercise in metaphysics in which the mind does not know it is philosophizing.'*"

That was one sentence... In the reading of it, I omitted certain interpolations, but I'm sure you nonetheless grasped the idea. To paraphrase, the idea being formulated here is that the world, as it appears to us, rests at a profound and fully abstract level on archetypal laws and that another formulation of the same laws is found in music - so that if one were ever in a position to comprehend fully the content, form and references of music, one would at the same time have comprehended fully the structure of the world. Schopenhauer explains this even more exactly when for example he attempts to relate the voices in four-part harmony to the natural world, equating the bass part with the mineral world, and so on. One could go into this more deeply and investigate more fully these connections, and one would obtain then perhaps a sense of why music produces the effect it does. Similar ideas also appear many times in the writings of the romantic philosophers.

I now make a big leap into our own century because the form in which we mostly know harmonics today is to a significant extent bound up with a man who lived in our own century and wrote many books about harmonics - "*The Textbook of Harmonics*" (original title: *Lehrbuch der Harmonik*) to name but one -- and that is Hans Kayser. Hans Kayser lived from 1891 to 1964. In the twenties, he instigated and also oversaw the publication by Insel-Verlag of a series entitled "*The Cathedral: Books of German Mysticism*" (*Der Dom: Bücher deutscher Mystik*). The series included a volume devoted to Kepler (although Kepler is not generally regarded as a mystic) and it was doubtless his study of Kepler's thought that led Kayser to develop his own ideas on harmonics. In Kayser's work, we find that his regard too is fixed once again on concrete natural phenomena: in one of his first books, for example, "*The Hearing Human*" (*Der hörende Mensch*) -- a new edition of which, incidentally, is now available[1] - he deals with harmonic laws in chemistry, crystallography, astronomy and botany.

In Hesse's The Glass Bead Game we find the following sentences: "*It was the achievement of one individual which brought the Glass Bead Game almost in one leap to an awareness of its potentialities, and thus to the verge of its capacity for universal elaboration. And once again this advance was connected with music. A Swiss musicologist with a passion for mathematics gave a new twist to the Game, and thereby opened the way for its supreme development. This great man's name in civil life can no longer be ascertained; by his time the cult of personality in intellectual fields had already been dispensed with. He lives on in history as Lusor (or also, Joculator) Basiliensis. Although his invention, like all inventions, was the product of his own personal merit and grace, it in no way sprang solely from*

1. Engel & Co Verlag, Stuttgart 1993 - Unaltered reprint of the Leipzig edition of 1932.

personal needs and ambitions, but was impelled by a more powerful motive. There was a passionate craving among all the intellectuals of his age for a means to express their new concepts. They longed for philosophy, for synthesis. The erstwhile happiness of pure withdrawal each into his own discipline was now felt to be inadequate. Here and there a scholar broke through the barriers of his specialty and tried to advance into the terrain of universality. Some dreamed of a new alphabet, a new language of symbols through which they could formulate and exchange their new intellectual experiences."

There has already been speculation as to whether by this Swiss musicologist Hans Kayser might not be meant. "*Das Glasperlenspiel*" was, after all, published in 1943, and Kayser, who emigrated in 1933 from Berlin to Switzerland, had by this time already published important works on harmonics. Kayser also wrote later to Hesse asking him to write a brief review of his book "*Akroasis*", a short introduction to harmonics, (English title: "*Akroasis: The Study of a Harmonics of the World*"). Hesse declined in friendly terms, mentioning however in the letter something about his reading of Kepler. There has been conjecture, indeed, Hesse has virtually been accused of having been swayed here by a sense of rivalry, as though there were something akin to a copyright on these ideas. I personally consider this suggestion absurd - I prefer the view that Hesse did not wish to be drawn more deeply into an association with Kayser - even though there may have been many similarities in their ways of thinking - because for him the time was not yet ripe for a concrete Glass Bead Game. The origins of this Glass Bead Game may date from our time but its actual development is reserved for a later epoch. Kayser's harmonics, on the other hand, is already very much a 'system' with its own axioms and theorems and presents - overly in many respects - the aspect of a closed system, as though everything were already "there" - and sometimes even slides over into the realm of ideology. To invite confusion between Kayser's "really existing" harmonics with the ideal vision of the Glass Bead Game would certainly not have seemed wise to Hesse - especially since, as his novel shows, he places a large question mark after the Glass Bead Game.

It is today still far too early for something like the Glass Bead Game to actually exist. All there are are certain approaches, and in the second half of this lecture, I would like to say something more concrete - more technical - about these approaches. I would like to present some of their foundations, so that statements such as Kepler's claim (cited earlier) to have encountered musical intervals in the movement of the planets can be understood. How are we supposed to conceive it? What are meant here by musical intervals? Musical intervals are normally heard as sounds. The planets emit no sounds.

To begin with, perhaps, a small aural exercise. Let's us listen simply to this sound :

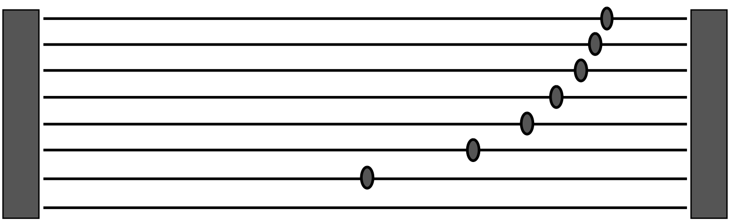
***Demonstration**: A monochord with many identically tuned strings is very gently and repeatedly strummed with the fingertips. A sound builds up in which, above the fundamental, many additional tones can be heard.*

All the strings of this instrument are tuned to the same tone, so you could say that only one tone was sounding. But I believe it could be heard clearly that there was not 'only one tone' sounding but a whole world of tones known as 'overtones'. And those tones are governed by certain laws and have a very definite structure. And with this instrument too, the second most important instrument in harmonics, one hears the same.

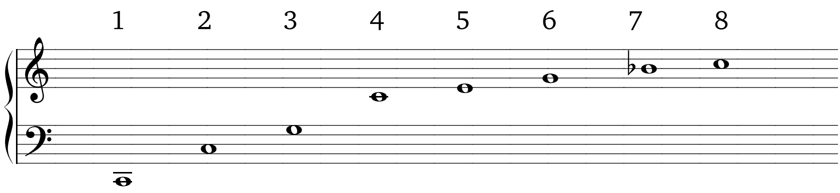
***Demonstration**: A length of hosepipe is held at one end and the other swung through the air. By varying the speed of rotation, a melody can be obtained similar to a bugle call or the tones produced by a herald's trumpet.*

Naturally that reminds us at once of something - a hunting horn, for example, or a herald's trumpet. Who put the herald's trumpet into the hosepipe? No one, obviously. It's a perfectly ordinary hosepipe; every hosepipe sounds like that. It is simply a natural law that is intrinsic to the vibrating column of air within it. And the fact that a herald's trumpet and a hunting horn sound the same way is due to the fact that these instruments have no valves and are therefore only capable of playing the various natural partials - it's the same with an Alpine horn. And what is particularly interesting is that in practice in this material - in the matter itself, in the law of the vibrating column of air or the vibrating string - there is something intrinsic that obliges us to say: "that is music". In other words, from this alone it is clear that music is not in fact something that human beings have thought up. There is naturally a continual interaction between cultural occurrences and that which emanates from Nature. But it is that to which Nature gives rise that constitutes the basis. And the music of other cultures - let us take, for example, Indian music or Chinese music - is very firmly based on these laws. The objection is often made: "Yes, but Indian music has completely different intervals - quarter tones and the like" but when you look more closely, it becomes clear that this is not the case. What we see is simply a cultural difference in the way the same material is handled.

We can now investigate where these tones lie on the string. For this purpose, I take this little round rubber disc with an incision reaching to the middle, which I can apply to a string of the monochord and slide it along. With it, I can prevent the string vibrating at a particular point, thereby creating a vibration node. If I place the disc at some arbitrary point on the string and begin to strum, the string can hardly vibrate and a strange muffled sound results. But as I slide it further along the string, a point is suddenly reached at which the string very clearly does vibrate. I leave the disc where it is and proceed in the same way with another disc on another string and try to discover whether other such points might exist. As I slide the disc further in the direction of the bridge, you will notice if you listen closely that there is a long succession of such tones, some quieter than others. And here, there is suddenly one that is a good deal louder than the others. Again, I leave the disc where it is, and take another disc and apply it to another string - and so on - until here we reach the eighth string. In theory, one could go on doing this for ever, but what you will surely already have noticed is that the tones became less distinct the higher they are. And what will also have been noted is the tendency of the curve. It is getting ever closer to the bridge but will never quite reach it. In other words, it approaches the bridge asymptotically; it's a hyperbole. And what law is at work here? As you will already have realized, these are the division points of the string: halfway along, a third of the way along, a quarter, a fifth, a sixth, a seventh, an eighth, a ninth, a tenth of the way along and so on. The sequence in fact stretches out to infinity, except that the higher you get, the more difficult the sounds are to make out.

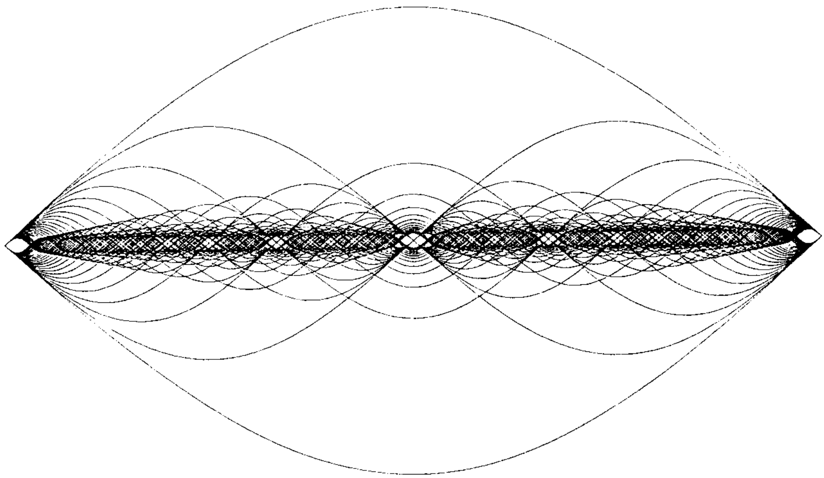


I will now play through once again, in order, the tones we have found:

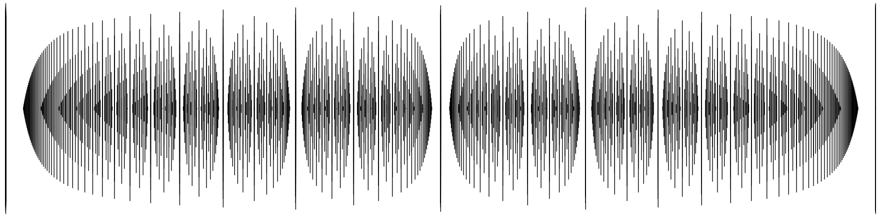


Note *on the display of tones in the written form of this lecture: It is in fact totally false to represent tones in this context as notes, since to do so presupposes that the notes and the tone system already exist. In reality what we are doing here is demonstrating phenomena that predated, and exist fully independently of, any tone system. This lecture is about experiencing the pure phenomenon outside of any cultural-musical context. Even the fundamental, C, has been arbitrarily chosen. The notes, therefore, only serve here to allow readers to play the tones through -- on a piano, for example -- in order to form an impression of what is meant, before (ideally) forgetting at once that they took the tones from the notes. Furthermore, the impression given by the tones written down and heard on the piano does not entirely correspond to the string division, since the piano is tuned somewhat differently. It would be best, then, to perform the procedure on a monochord.*

We recognize here too the sequence that I played just now with the hosepipe or that we heard earlier from the open strings. We have, then, what we call 'the overtone series', which is composed of quite specific musical intervals. Perhaps I should set them once more using fixed bridges so that you can hear them better. If I now position bridges underneath the strings, I must measure, how much is a third, how much is a quarter. This represents an important difference from the other technique of looking for the tones with the rubber discs, because there, if I am not exactly at the point of division, there's simply no sound. And I can identify the halfway point, for example, because it's the place where, when the disc sits there, the best sound is obtained. It's not like that with the fixed bridges. When I move them, the string no longer sounds as a whole, but rather, the bridge forms virtually in every position a new beginning for the vibrating string and there is therefore no point that particularly stands out. It also means that I have to measure how long this section of string is in relation to the whole in order to set the tone. Two completely different procedures, then. Perhaps two diagrams will make the point more clearly. You can also picture the string as vibrating within itself; not only does the entire string vibrate, but it vibrates also in sections - i.e. once as a whole string, then in two halves, three thirds, four quarters, five fifths etc.



Or another picture. This one shows where all the divisions would lie on the string. Or, putting it another way, it shows how all the fractions or rational numbers are distributed along a unit length. This is shown here down to a hundredth. And you can see quite clearly that in the area around the middle, there are no other divisions. It is almost as if the higher numbers were according the lower numbers, the two, the three, the four, more room - out of respect for the value of the small numbers. You can hear the same phenomenon if you move the rubber disk about when it is near the midpoint of the string. Just before you arrive at the very middle, you can hear the various tones becoming ever more faint, and then at a certain point, around one centimeter from the middle of the string, you get into its catchment area, into this gap, and this very powerful tone then asserts itself.[2]



Let us listen, then, once again to the overtone series. Here the series extends to the eighth partial. I say 'partial': I should explain here something about the terminology. We speak of overtones and of partials. In principle, these are the same thing, except that, when numbering them, you have to be careful: if you're talking about overtones, you don't include the fundamental, so dividing the string in half gives you the first overtone, dividing it in three, the second overtone, and so on. That, however, is somewhat inelegant: the numbering does not correspond with the number that produces the tone. For this reason, it is better to talk about partials, where the fundamental is included, so half the string produces the second partial, a third of the string, the third partial and so on. The number of a partial, in other words, not only identifies it but also reveals something of its essence.

Let us listen then, one after the other, to the intervals produced by these numbers. Going first from the whole string to the half, the ratio is ***1 : 2. (Where in the following text numbers appear, during the lecture the corresponding strings are plucked; the reader should do the same on an instrument following the numbering of the notes given above)***. When anyone familiar with the musical context and terminology hears this interval, they will perhaps say: 'aha!' - and think at once of the conventional term for this interval. That is not, however, necessary and is perhaps even harmful. What is important is the phenomenon itself, and if a name for it comes immediately to mind, it is possible that you will forget to wonder about it, and you cannot wonder enough about it. What does this interval *do*? You could say, it is an interval and yet it is

2. A reproduction of this image (1.2 meters in length) to be laid beneath the monochord is enclosed with the book.

not one. It is, naturally, an interval, because it represents a distance between two tones, and yet it creates no new interval quality. Musically, it is called an octave, but this name in itself in fact tells us nothing. If, for example, we call the fundamental C, then the tone lying an octave higher - i.e. in the ratio ***1 : 2*** - is again C, and the same is true of all the octaves above. The same tone, then... what does that mean: "the same tone"? I think very few people have really thought about what they mean when they say 'it is the same tone'. The statement is, after all, totally absurd and paradoxical. You in fact have a quite different tone, these tones are quite obviously different and are in fact a long way apart, and yet you say nonetheless that both are, for example, "C". The same tone, and yet a different one. The whole of music in all cultures is based upon the octave, which is therefore something we take for granted. When, for example, men and women sing together the same melody, the men usually sing an octave lower than the women, and yet we say they are singing in unison: that they are singing "the same thing". That means we regard the octave as the same thing as a matter of course, even though it is in fact a different tone, and yet no one wonders about it.... One could philosophize at length on this subject, and it is - I find - truly difficult to grasp.

I have been calling this tone "C". I should say something else at this juncture to avoid confusion. It is important, namely, to bear in mind that it is not the tone C on the piano that is meant here nor any other pitch in particular. It is often a source of error in music or harmonics that two completely different things get confused: absolute pitch, on the one hand, and intervals or relative pitch on the other. I might sing, for example, a tone - any tone - remember its pitch, go to the piano and establish: "ah, it was a G". That this tone is a 'G' in fact tells us nothing about the tone I actually sang. If I had been alive a few hundred years ago and gone to the piano, then the same tone would perhaps have been an A or something else, because in those days pianos were tuned to a lower pitch; in fact, in those days, there was a 'chamber pitch' and a 'choir pitch', which were different, and different cities even had differing chamber pitches. Of course, for this reason at a later date chamber pitch was defined by agreement in terms of vibrations (or cycles) per second, whereby A, for example, was set at 440 Hertz. This standard, like the meter in Paris, allows a flautist in Berlin to play in Munich without having to saw a bit off his flute: very practical. The octave, on the other hand, has never needed to be standardized, because everyone can hear it...

On the other hand, when earlier I called our fundamental "C", I wasn't referring to a tone with a particular pitch; I simply called it C because in our musical system I am accustomed to think of tonal relationships as being most simply expressed if one begins from C, but C here is merely another name for the fundamental and it is of no importance what the absolute pitch of this tone is. Or, better put, it has no pitch because it is only the point of reference for an abstract system of relationships; in order to make these relationships audible, however, I obviously have to play them at some pitch. It would be the same in geometry, if I were thinking of two line segments, one twice as long as the other, the absolute lengths being irrelevant: if drew them here on the projector, their lengths might be five and ten centimeters, whereas their projected images on the wall might be one and two meters respectively. In Indian music, for example, this fundamental is called "*Sa*", and even that is simply a shorter name for "fundamental of the scale", and it has no definite pitch there either. All of which simply by way of explaining the double use of the tone names.

But back to our string division: by halving the string length, we have discovered not only that the half sounds an octave higher but also a law. That is to say, we know now in advance what tone will sound when the string length is reduced to a quarter and an eighth of the original, since a quarter is half a half, and an eighth half a quarter. In other words, we have discovered a law stating that however many times we halve the length of the string, the result will always be the same tone - or something we experience as being the same tone - only in a different register.

Octave Octave Octave

1 : 2 : 4 : 8

c c’ c’’ c’’’

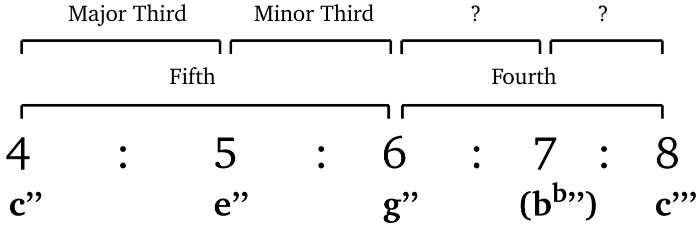
Let us listen to the next tone, namely the ratio ***2 : 3***. Now we can hear very clearly a quite different tone. If we compare it one more time, what a feeling, what an experience it is - on the one hand ***1 : 2***, and on the other ***2 : 3***, the new interval: that is quite clearly a different experience, musically one describes it as a fifth. But that is not in fact important - names here mean nothing - or even worse, they are even confusing: here, the word '*fifth*' might lead you to suppose that the interval had something to do with the number *Five*, but it hasn't. Because we have heard that the interval known as a fifth is produced by dividing the string into *three* parts. That means we can say the number *Three* produces this quality and this experience. The only reason we call it a fifth is because it is the fifth tone of our familiar scale. If the name had originated in a culture where, for example, twelve tone music was the norm, then the fifth would not be called a fifth but an octave, because it is the eighth tone of the twelve tone scale. In other words, these names, such as octave, fifth etc. are in fact completely arbitrary or, better put, culturally determined. If, on the other hand, we were to call the fifth "*Three*", then the essence and the name would correspond, because it is the *Three*, in fact, that produces this interval.

Now it is in fact not entirely correct to say that the number *Three* produces the fifth, because it produces admittedly a new tone, if we start from C then a G, but this new tone produces two new intervals or relationships ***2 : 3*** and ***3 : 4***. The *Two* and the *Four* we have already had as the octave space. Within it, the *Three* now makes an appearance and enters into a relationship with the neighboring tones, from C to G and from G to the C above, which is not the same experience as is we were simply to return to the C below. If one investigates these qualities, one might perhaps say, this interval ***2 : 3*** has an element of coming-out-of-oneself, of opening-out. It is as if the fundamental were putting this new tone outside of itself. And if one comes then from this number *Three* back to *Four*, which is in fact the fundamental again at a higher level, one has the feeling, that one is returning. It has a resolving quality, it is like an affirmation, a confirmation. This experience, the affirmative interval ***3 : 4*** is musically the fourth. The *Three* enters into the octave and produces the fifth and the fourth, and one could say, the fifth and fourth are the two aspects of the number *Three*. It is also interesting to observe that *Three* is the arithmetic mean of *Two* and *Four*: the difference between *Two* and *Three* is exactly one and that between *Three* and *Four* is also exactly one, that means, I must take equal steps to move from the one number to the other. Arithmetically, the *Three* therefore divides this space equally. Musically, however, it divides it unequally into the aspects fifth and fourth - it does not simply divide the space, it polarizes it. This always reminds me of the story of the creation: "So God created man... male and female created he them".

Octave		Fifth		Fourth		Fifth		Fourth	
1	:	2	:	3	:	4	:	6	: 8
c		**c’**		**g’**		**c”**		**g”**	**c’’’**

What the number *Six* is, we also already know because the ratio of ***2 : 3*** is, of course, the same as the ratio ***4 : 6***. In other words, that, too, must again be a G, or from the *Four* to the *Six* also a fifth again, and from the *Six* to the *Eight* also a fourth again.

The question that arises now is: can we also tell in advance what the *Five* will be? With the *Six*, we were able to deduce the answer from what we knew already. And if one thinks it over, one realizes: no, we cannot know that. And that is something totally fundamental, that one must first hear these numbers so that they become an experience, and it is the case for each new prime number that does not arise from other numbers. Let us listen to that now: first the fifth ***4 : 6*** that we already have and then the *Five* between the two: ***4 : 5 : 6***. Here, once again, a totally new experience, a new feeling. Into this empty fifth comes the *Five* and does something - what does it do? Hard to describe, naturally, but perhaps one can express it like this: that here something human, something subjective, downright sentimental perhaps or in the extreme case kitsch appears. If I use pejorative terms here, it is not to accuse the number *Five*, but the possibilities of its spectrum do permit of such language. When a musician hears it, he says: aha, so the ratio ***4 : 5 : 6*** is the major triad, and finds that perhaps frightfully trivial. The triad is however the element upon which for five hundred years our entire music has been based. But five hundred years ago, people did not at all experience it as trivial - nothing short of euphoria is what the number *Five* produced at that time in music. The *Five*, then, enters the empty fifth and creates a new quality, and this quality, too, has two aspects: the major third ***4 : 5*** and the minor third ***5 : 6***; the *Five* therefore polarizes the fifth in a manner analogous to that in which the *Three* earlier polarized the octave.



We already know of the *Seven* that it is a prime number and that we must therefore listen to it in order to experience whatever new quality it brings. Let us listen then: first to the pre-existing triad ***4 : 5 : 6*** and then to the *Seven* added ***4 : 5 : 6 : 7***. Very clearly, then, there is also something new here; one has perhaps the feeling that the *Seven* puts a question, that it puts a question to the triad. The number *Seven*, by the way, was previously almost unused in music - it sounds similar to the dominant seventh in our music, but only similar, and it also has a quite different function. To address the musicians: it is not a *dominant seventh* but a *tonic seventh*, and it is only today that it has found its own way in this role into our music, in blues and in jazz, although attempts were made by a few musicians two hundred years ago to integrate it into the musical system. I think it will take some time yet before one really knows and can feel what this *Seven* really means. In the case of the higher prime numbers, the *Eleven*, the *Thirteen*, it becomes ever more difficult from hearing and experiencing to obtain access, precisely because they still belong far less to our internal musical vocabulary - one can only approach these numbers very tentatively. Only as a suggestion: perhaps the *Eleven* has something to do with the emancipation of the tritone, and the *Thirteen* with the golden ratio - but we must leave that open and it does not contribute very much in the present context since one cannot yet experience it vividly.

I think we will have to content ourselves with that for now. We do not need to investigate the technical side in much detail. We would prefer to offer a few fundamental thoughts on what we are actually doing here. We are talking here about numbers, and numbers of course are primarily associated with something purely technical, purely quantitative. That is to say: one measures something, one counts something, one weighs something, and with normal science it is generally the case that everything is quantified, and the number is an aid to quantifying things. Now we suddenly realize that a number not only has a *quantitative* aspect but that it also has a *qualitative* aspect. We experience this when we hear the number, and here we are hearing in fact the number itself. I hope it has become clear what the difference is. If I say, for example, "the fifth is the fifth tone", could I not also say, "I am hearing the *Five*?" No, I am not hearing the *Five*, but hearing the *Three*. That means, in the one case the number is intrinsically bound up with a definite phenomenon, and in the other case, when I use the name 'fifth', then the number five, which is contained within it, is simply a name, a road sign as it were, a cultural convention. But what we have heard here are the actual numbers themselves. I would even say that the qualitative side of the numbers is their primary characteristic, and the fact that they can be used to express quantities merely an ancillary one.

We have therefore a connection between a *quantitative* and a *qualitative* side of the same thing. And when you have concerned yourself with this qualitative side of the number for a time, above all through listening, but also through geometry and similar paths, then the numbers, which turn up everywhere, say something quite different to you than they did previously; structural and content-related connections surface. Here one must naturally be careful: most frequently when we encounter numbers in everyday life, they are in fact only meant quantitatively, because that is the use people most often make of them. If, for example, the address of this house is 29 Ismaninger Street, the number 29 here has no quality because it is simply being used to number the houses of the street. There are people, of course, who go in for kabbalism and want to extract from this number some other significance, but that is a totally different matter of no relevance to our subject. The same is the case when numbers are used as quantifiers; whether there are seven or nine cars in the street naturally has no qualitative aspect either.

So, what type of manifestation of the numbers is left to be relevant for our consideration? The qualitative character of the numbers always appears when the number determines a structure or a structure exhibits a number. That begins for example with the atomic structure of the elements, where numbers determine that the structure of the elements is so and not otherwise and with it determine also their chemical properties. It continues with the chemical ties of the elements, which then also determine the structure of the crystals - the number *Five*, for example, almost never appears in the mineral world, only the numbers *Three*, *Four* and *Six*, as one sees so beautifully illustrated by snow crystals. The number *Five* makes its first appearance with plants - there are, of course, many plants with fivefold symmetry, such as for example the plants of the rose family (rosaceae). In the case of the rose itself, one does not come so easily upon it, because the cultivated varieties have so many petals; you have to examine its pistil to discover the *Five*. But most fruit trees belong to the rose family, and if you cut through an apple horizontally, you can see very clearly the five-pointed star. Other plants, such as the alliaceae, the lily, for example, are constructed upon the number *Three*. There we have a clear opposition between the rose with the *Five* on the one hand and the lily with the *Three* on the other, and if one has obtained a sensory impression of the numbers *Three* and *Five* through one's consideration of the tone numbers, then one can make a start with this evidence. With animals, too, above all the lower animals, such as starfish and sea-urchins, we come across fivefold symmetry; among the higher animals and men, dual symmetry occupies the foreground.

One might say, then, that the rose expresses itself through the number *Five*. I, however, would formulate it the other way around: the number *Five* expresses itself through the rose, the rose is one form in which the number *Five* manifests itself. It is perhaps not customary to think of it that way - but I also stated earlier that the *Five* "does" something, namely produces the experience of the third. In this way, the totality of all forms of manifestation of a number in geometry, music, nature and culture comes by its "face", the number can be understood as a fundamental force, the physiognomy of which can be read everywhere. Expressed in mythological terms, one could also say that the numbers are gods whose essence is manifested in the world. It is the case in all mythologies that we have great gods, main gods and a multitude of lesser gods that lose themselves in endless ramifications. With the numbers, it is correspondingly so: the small numbers are the ones with the greatest archetypal force, and the higher you go in the series of numbers, the more difficult it is to differentiate them in terms of their force or character. Numbers also appear regularly in the symbolism of all cultures and with this tool it becomes possible to investigate possible common fundamental structures even in cultural phenomena.

I believe it has become clear to some extent what composition the individual glass beads of our game might have. I have, of course, in the introduction to the program of our harmonics events written that it is our goal to develop the building blocks of this game. That seems to me, the more I think about it, to be setting our sights somewhat high, because I keep asking myself whether this game will ever really exist in reality. Perhaps the whole thing is simply a vision of men who, due to a specific inner structure, have to think in that way, the vision of a paradise of which one does not know whether it can really exist in this world - and I am tempted to say, that I consider it unlikely. But perhaps the important thing, as is so often the case, is to travel - which is enough - rather than necessarily to arrive. There is in Hesse's '*The Glass Bead Game*' a poem that expresses it all beautifully: the journeying, the fascination, the surmise... And behind all of these, a large question mark:

A Dream

Guest at a monastery in the hills,
I stepped, when all the monks had gone to pray,
Into a booklined room. Along the walls,
Glittering in the light of fading day,
I saw a multitude of vellum spines
With marvelous inscriptions. Eagerly,
Impelled by rapturous curiosity,
I picked the nearest book, and read the lines:
The Squaring of the Circle - Final Stage.
I thought: I'll take this and read every page!
A quarto volume, leather tooled in gold,
Gave promise of a story still untold:
How Adam also ate of the other tree...
The other tree? Which one? The tree of life?
Is Adam then immortal? Now I could see
No chance had brought me to this library.
I spied the back and edges of a folio
Aglow with all the colors of the rainbow,
Its hand-painted title stating a decree:
The interrelationships of hues and sound:
Proof that for every color may be found
In music a proper corresponding key.
Choirs of colors sparkled before my eyes
And now I was beginning to surmise:
Here was the library of Paradise.
To all the questions that had driven me
All answers now could be given me.

Here I could quench my thirst to understand,
For here all knowledge stood at my command.
There was provision here for every need:
A title full of promise on each book
Responded to my every rapid look.
Here there was fruit to satisfy the greed
Of any student's timid aspirations,
Of any master's bold investigations.
Here was the inner meaning, here the key,
To poetry, to wisdom, and to science.
Magic and erudition in alliance
Opened the door to every mystery.
These books provided pledges of all power
To him who came here at this magic hour.

A lectern stood near by; with hands that shook
I placed upon it one enticing book,
Deciphered at a glance the picture writing,
As in a dream we find ourselves reciting
A poem or lesson we have never learned.
At once I soared aloft to starry spaces
Of the soul, and with the zodiac turned,
Where all the revelations of all races,
Whatever intuition has divined,
Millennial experience of all nations,
Harmoniously met in new relations,
Old insights with new symbols recombined,
So that in minutes or in hours as I read
I traced once more the whole path of mankind,
And all that men have ever done and said
Disclosed its inner meaning to my mind.
I read, and saw those hieroglyphic forms
Couple and part, and coalesce in swarms,
Dance for a while together, separate,
Once more in newer patterns integrate,
A kaleidoscope of endless metaphors -
And each some vaster, fresher sense explores.

Bedazzled by these sights, I looked away
From the book to give my eyes a moment's rest,
And saw that I was not the only guest.
An old man stood before that grand array

Of tomes. Perhaps he was the archivist.
I saw that he was earnestly intent
Upon some task, and I could not resist
A strange conviction that I had to know
The manner of his work, and what it meant.
I watched the old man, with frail hand and slow,
Remove a volume and inspect what stood
Written upon its back, then saw him blow
With pallid lips upon the title - could
A title possibly be more alluring
Or offer greater promise of enduring
Delight? But now his finger wiped across
The spine. I saw it silently erase
The name, and watched with fearful sense of loss
As he inscribed another in its place
And then moved on to smilingly efface
One more, but only a newer title to emboss.
For a long while I looked at him bemused,
Then turned, since reason totally refused
To understand the meaning of his actions,
Back to my book - I'd seen but a few lines -
And found I could no longer read the signs
Or even see the rows of images.
The world of symbols I had barely entered
That had stirred me to such transports of bliss,
In which a universe of meaning centered,
Seemed to dissolve and rush away, careen
And reel and shake in feverish contractions,
And fade out, leaving nothing to be seen
But empty parchment with a hoary sheen.
I felt a hand upon me, felt it slide
Over my shoulder. The old man stood beside
My lectern, and I shuddered while
He took my book and with a subtle smile
Brushed his finger lightly to elide
The former title, then began to write
New promises and problems, novel inquiries,
New formulas for ancient mysteries.
Without a word, he plied his magic style.
Then, with my book, he disappeared from sight.

About this article

All the articles are revised versions of lectures delivered in the context of events organized by the "Arbeitskreis Harmonik" (Study Group Harmonics) in the Freies Musikzentrum, Munich.

Peter Neubäcker: **Harmonics And The Glass Bead Game**

Lecture delivered on the 13th March 1993 as an introduction to harmonics. Since the lecture featured numerous demonstrations on the monochord among other things, the lecture was rewritten for the book

Peter Neubäcker

Born on the 26th June 1953 near Osnabrück. After leaving school, a period of intensive searching for inner ways culminating in a three-year sojourn in a spiritual community. After that, apprenticeship as a violin-maker by Lake Constance. '79 onwards, own instrument-making workshop in Munich. There also three years' training as a non-medical practitioner, parallel to which, training in astrology. Development of astronomic-astrological materials that led to the founding of own publishing house that also forms my living basis (from a material point of view) along with courses in guitar-making and computer music at the Freies Musikzentrum in Munich.

For almost twenty years the emphasis of my work as regards content is the search for correspondences between structure and content - mainly in the field of music but also with reference to the natural sciences and philosophy - mostly and wherever possible formulated mathematically. From which it follows that I am engaged most of the time (alongside, or rather over and above, the work mentioned earlier) with musical-harmonic research and algorithmic composition and lead in terms of organization and content the "Arbeitskreis Harmonik" in Munich.

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