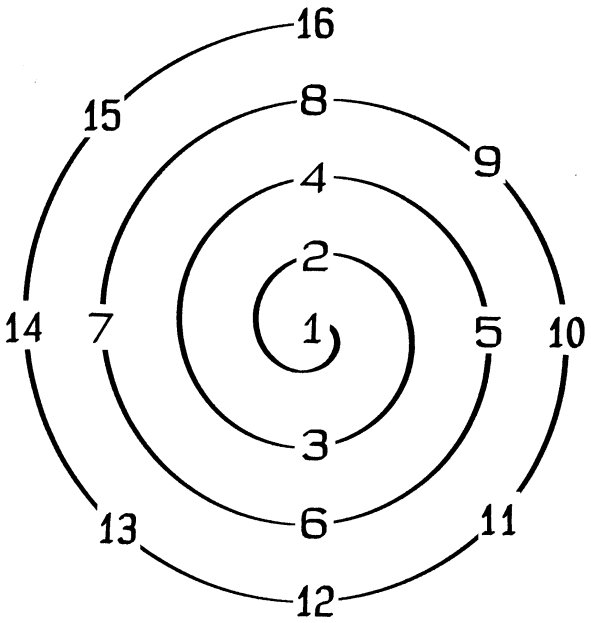
HARMONICS AS SYMBOLISM



Peter Neubäcker
Arbeitskreis Harmonik

CONTENTS

Dedicated to
Prof. Dr. Rudolf Haase
Founder of the Hans Kayser Institute
in Vienna For Basic Research In Harmonics
on the occasion of his 70th birthday on the 19.2.1990

Expanded transcript of the lecture
„Symbolische Inhalte der mathematisch-musikalischen
Beziehungen in der Pythagoräischen Harmonik"
delivered during the symposium „Kult-Mythen-Symbole"
28.9.-1.10.1989 in Brixen

PART 1: WHAT IS 'HARMONICS'?

The Origins

When one speaks of Pythagorean harmonics, it is not 'harmonics' in the sense of overtones nor is it the theory of harmony in music that is to be understood; but rather the meaning of the word goes further back to the Greek word 'harmotto' – which means something like 'to ordain' or 'order': one is looking, then, here at how things relate to each other and are ordered, and the assumption is that this order can be expressed in musical terms. The primary significance of the epithet 'Pythagorean' here is not something historical, something relating to the teachings of Pythagoras, but serves to characterize more closely a particular way of considering things – when one speaks of Platonic ideas, one does not mean the ideas of Plato either, of course, but a very specific characteristic way of dealing with the world of ideas.

With harmonics, it is also about something akin to Platonic ideas, that form the subsoil of music, and as we shall see, not only the subsoil of empirical music: Music is understood here as something that makes audible and crystallizes archetypes or ideas, as world-matter, with which the musician plays and with which the soul deals when man hears music.

To be able to understand these connections better, we must enter a territory in which philosophy moves between mathematics and music. At first sight, the involvement of mathematics may seem unfamiliar and contradictory, but if one looks back in the history of ideas, one realizes that there were times in which this involvement was considered perfectly natural. I should like here to provide a small overview of how these ideas, which lie at the heart of what we today call harmonics, developed historically.

Men in earlier times had – and still may have today in primitive cultures – a natural sense that the entire world in its structure was somehow related to music. The creation myths of many peoples tes-

tify to this – the Indian myth, for example, that the Creator-God through singing caused the lump of earth in his hands to grow – as does the fact that the Polynesians use the same word for 'world' and 'singing': langi – the universe and lalolangi – the lower singing: the Earth.

For our cultural area there is in ancient Greece the cult of Orpheus, in which music played the central role – as a power that stemmed from the source of the world. In all these old cultures it was therefore clear that music is a creative elemental force – to the magical-mystical consciousness that was a very natural feeling.

Now around two and a half thousand years ago a new element appears: the burgeoning rational thought is no longer satisfied with the instinctive feeling that the world is based on a musical structure – man also wants to know what this structure is like. The key figure for this process in Greece is Pythagoras. He had spent the first half of his life as a student of the mysteries in Egypt and Babylon, where he also studied the further developed mathematics there before returning to the Greek cultural area and founding in Croton in Southern Italy a school of philosophy and an order.

Nothing written by Pythagoras himself has been handed down to us, but the influence of his thought in the period that followed was so great that traces of his teachings, from which his world view can be deduced, are to be found everywhere. In Plato's Dialogues, for example, Pythagoreans appear who represent his cosmogony: here the creation of the world appears as at once a mathematical and a musical process, and Aristotle, too, says of the Pythagoreans: "since... they saw that the modifications and the ratios of the musical scales were expressible in numbers; - since, then, all other things seemed in their whole nature to be modelled in numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a scale and a number."

What is new here is the idea of the mathematical structure of the world that has been added to the musical, and also new is that these

connections were not taught as articles of faith but the Pythagoreans conducted active research in an attempt to learn more about these structures of the world. The instrument of research that Pythagoras introduced for the purpose was the monochord: a single-stringed instrument not primarily intended for making music but for conducting measurements. With its help, the Pythagoreans were able to establish that "... the laws of musical harmony can be represented in numbers..." If, in other words, the string of the monochord is divided in such a way as to create a euphonious musical interval, it emerges that the divisions of the string form a particular numerical ratio, and the smaller the numbers involved, the more pleasant the sound of the interval. The first two numbers, for example, the One and the Two, yield the octave, the foundation of all music, and from the numbers Two and Three we obtain the fifth, from which all the other notes can be generated.

One can say that with the monochord Pythagoras introduced the first scientific instrument of experimentation, and even today the monochord remains the most important tool of harmonicists. The monochords of today generally have multiple strings all the same length and identically tuned so that by placing small bridges beneath the strings, it is possible to set and hear different numerical ratios at the same time.

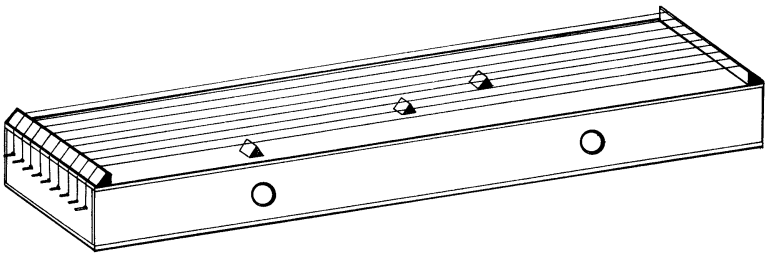


Figure 1: A monochord

One important characteristic, however, distinguishes the monochord from what we would think of today as a scientific instrument of experimentation: through the fact that one is on the one hand dealing with numbers but on the other hand experiencing these numbers directly through hearing it connects these two worlds with one another: the inner world of experience with the external world of measurement and calculation – it creates a link, in other words, between the qualitative and the quantitative.

Of Pythagoras it is also said that he was the first to describe the world as a 'cosmos' – and cosmos means 'order' or 'decoration'. This word is therefore related in meaning to the term 'harmonics', since this is to be understood as derived from 'harmotto' in the sense of 'order' – harmonics, in other words, is a study of the structures of the whole world – and the practice of 'listening to them' on the monochord.

To the ancients, then, the musical foundation of the world was a premise they took for granted – what was new was the idea that this could be grasped rationally through numbers. Through the vivid association of the number with the sound, the numbers were not, as for most people today, dead abstract constructs but living beings whose character and quality could be experienced through hearing.

The Pythagoreans comprised the Mathematici and the Acusmatici – the word Mathematici here, however, is not to be understood as 'mathematicians' in today's sense. The Mathematici were those who attempted to penetrate the essence of things through their own research as opposed to the Acusmatici, the "Listeners", who adopted the teachings of Pythagoras as maxims to live by.

As is usually the case, a system of statements and rules survived from the teachings of Pythagoras that the 'Acusmatici' of later times scarcely understood correctly but to the 'Mathematici' in the Pythagorean sense an impetus was given that even today lives on in harmonics.

From Pythagoras onwards, the connection between music and mathematics was familiar to all later philosophers and mathematicians (which were still one and the same at that time) and an important foundation of thinking and research. The mathematician Euclid, for example, also wrote a book about music and its mathematical foundations and the astronomer and geographer Ptolemy describes in his musical work studies with a 15-string monochord as still used today.

Johannes Kepler – The rational way

The Pythagorean world view, therefore, was alive throughout the classical era and middle ages, with the ideas of the Harmonia Mundi and Harmony of the Spheres particularly popular. But in the scholastic philosophy of the Middle Ages, one began primarily from thought itself, and philosophical investigation proceeded more geometrico – in the manner of geometry. The Harmony of the Spheres had become a fixed part of the intellectual edifice, and no one asked about the manner in which this harmony – in the movement of the planets, for example – manifested itself in the external world.

It was not until the Renaissance era – around the 16th century onwards – that research began once again to examine external nature. For harmonic research, Johannes Kepler (1571-1630) is the most important figure here. Since Kepler's thought and research is especially characteristic of the manner of thinking of the new harmonics, I should like to discuss him in greater detail.

From his youth onward, Johannes Kepler was convinced that the entire world was harmonically ordered according to particular laws, and unlike the attitude of mind of the ancients he was also of the opinion that this order could be discovered through research into external nature. When today in school we learn Kepler's three laws of planetary motion, we are not normally told that these laws were for him simply a by-product of his search for the musical harmony of the world, which he then also discovered in fact.

Kepler's formulation of the question was an unusual one compared with the approaches to natural research today: from the laws of heavenly mechanics known to us today one can admittedly calculate how far away from the sun a planet is throughout an observed revolution period – but as to why it is exactly this distance and has this revolution period rather than another, there are no clues. It was just this question that interested Kepler and this manner of contemplation is typical of the harmonic way of thinking: It is a question therefore of discerning correlations with a perceptible character that contain a 'physiognomic' testimony for the observer; the functional relationships are of secondary importance. The question is similar to that of why the Rosaceae are constructed upon the number Five and the Liliaceae upon the number Three: modern science is incapable not only of answering such questions but even of putting them, since contemplation of this kind has no place in its system of thought.

Initially Kepler's search for the ordering principle of the solar system rested on an idea that was more geometrical than musical: At the age of 23, when he was a professor of mathematics in Graz, the idea came to him in the middle of a lecture that the structure of the solar system might be based on the geometry of the Platonic solids. These are the regular polyhedra that were described for the first time by Plato: if you attempt to construct geometric solids the faces of which are identical regular polygons that meet at the same three-dimensional angles, it turns out that only five are possible: the best known is the cube, which is constructed from six squares; from equilateral triangles it is possible to construct three such polyhedra; the tetrahedron with four, the octahedron with eight, and the icosahedron with twenty faces; the last of the Platonic solids is the dodecahedron, which is formed from twelve pentagons. The five solids are shown in Figure 2, a page from Kepler's later work Harmonices Mundi: superimposed here on each of the five polyhedra is a drawing representing one of the elements (fire, water, air, earth and the cosmos or 'quinta essentia') – the assignments being the same as those found in Plato.

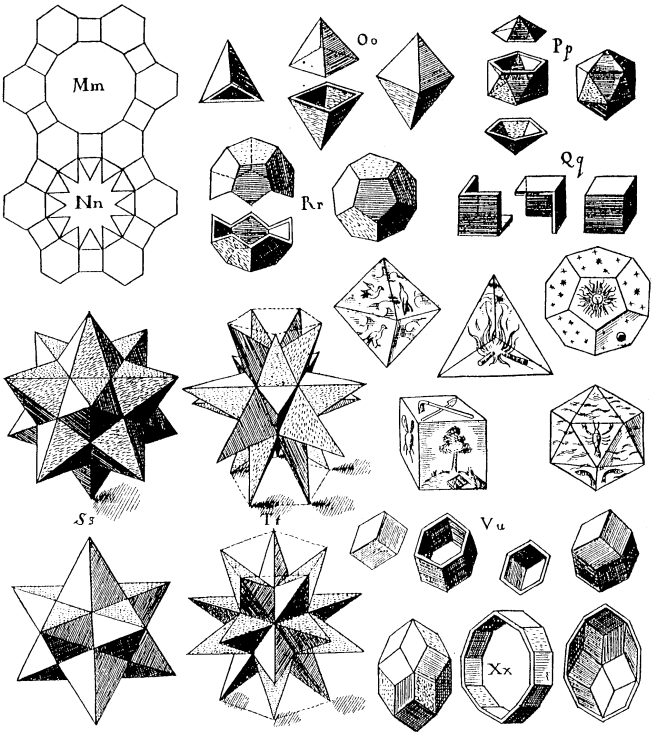


Figure2 : From the 2nd volume of Weltharmonik

These five solids have the characteristic that one can inscribe them with a sphere that touches all their faces from the inside and circumscribe them with a sphere that touches all their vertices from the outside. If you now nest the five bodies in such a way that the outer sphere of one polyhedra is the inner sphere of the next, you arrive at the model illustrated in Figure 3. The resulting sequence of radii corresponds with astonishing accuracy to the distance of the orbits of the planets from one another. Kepler first published this discovery in 1596 in his work Mysterium Cosmographicum of which he sent a copy inter alia to Galileo Galilei.

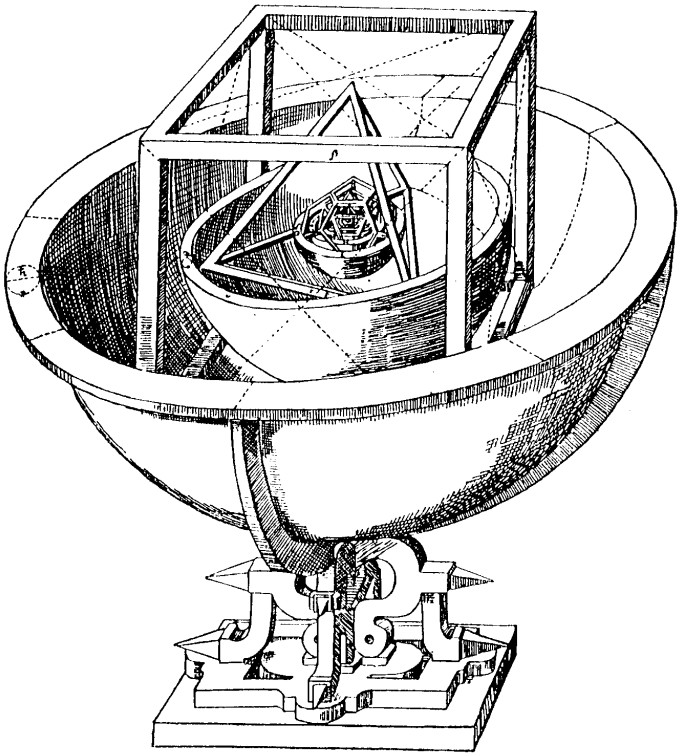


Figure 3: Kepler's Mysterium Cosmographicum

The Music of the Spheres

Admittedly the outer planets, of which the first, Uranus, was not discovered for another two hundred years, have no place in this model, for the simple reason that no more than five such solids are possible, and the exactitude of the correspondence did not satisfy Kepler himself either. This discovery, however, gave him the enthusiasm and impetus to continue searching for evidence of the harmonic structure of the world. This search also led him to Prague, to the astronomer Tycho Brahe, who at the time had the best astronomic

observation results. Brahe died only a year later and Kepler became his successor as imperial mathematician in Prague. Here he had an opportunity as a result of the observations of the heavens at his disposal to continue his research, developing in the process his well-known planetary laws. These, however, to him were nothing more than minor components of his most important work: Harmonices Mundi – the five books of world harmonics, which he published in 1619 in Linz.

In this work, he presented his view of world harmonics: in the first two books, he derives the qualities of the numbers from geometry, translating them in the third book into the audible realm on the monochord and developing a comprehensive musical theory; in the fourth book, he applies the discoveries from geometry and music to astrological observation. In the fifth book, the astronomical part, he presents his fundamental harmonic discovery: the realization of the musical intervals through the movement of the planets. Having first investigated the distances and periods of revolution of the planets from different viewpoints and arriving at no satisfactory results, he considers finally the angular velocity of the planets as they would be perceived by an observer on the sun and arrives at the following table:

Harmonies between each pair of planets			Apparent daily movements				Intrinsic harmonies of the individual planets				
Diverg.	Konverg.				′	″					
$\frac{a\ 1}{d\ 3}$	$\frac{b\ 1}{c\ 2}$	Saturn	Aphel	1	46.	a	Between	1	48	$\frac{4}{5}$	Major third
			Perihel	2	15.	b	and	2	15		
$\frac{c\ 1}{f\ 8}$	$\frac{d\ 5}{e\ 24}$	Jupiter	Aphel	4	30.	c	Between	4	35	$\frac{5}{6}$	Minor third
			Perihel	5	30.	d	and	5	30		
$\frac{e\ 5}{h\ 12}$	$\frac{f\ 2}{g\ 3}$	Mars	Aphel	26	14.	e	Between	25	21	$\frac{2}{3}$	Fifth
			Perihel	38	1.	f	and	38	1		
$\frac{g\ 3}{k\ 5}$	$\frac{h\ 5}{i\ 8}$	Earth	Aphel	57	3.	g	Between	57	28	$\frac{15}{16}$	Semitone
			Perihel	61	18.	h	and	61	18		
$\frac{i\ 1}{m\ 4}$	$\frac{k\ 3}{l\ 5}$	Venus	Aphel	94	50.	i	Between	94	50	$\frac{24}{25}$	Diesis
			Perihel	97	37.	k	and	98	47		
		Mercury	Aphel	164	0.	l	Between	164	0	$\frac{5}{12}$	Octave with minor third
			Perihel	384	0.	m	and	394	0		

Figure 4: Table of angular velocities

Since according to Kepler's discovery the orbits of the planets are not circular but elliptical, the velocity of each differs at its closest and most distant points from the sun (denoted by perihelion and aphelion respectively in the table) – if you now set these velocities as string lengths on the monochord you obtain the musical intervals – both when comparing the two velocity extremes of each single planet and when comparing the movements of different planets with one another.

Euphoric over his discovery, Kepler wrote: "Now because 18 months ago the first dawn, 3 months ago broad daylight but a very few days ago the full sun of the most highly remarkable spectacle has risen — nothing holds me back. I can give myself up to the sacred frenzy, I can have the insolence to make a full confession to mortal men that I have stolen the golden vessels of the Egyptians to make from them a tabernacle for my God far from the confines of the land of Egypt. If you forgive me I shall rejoice; if you are angry, I shall bear it; I am indeed casting the die and writing the book, either

for my contemporaries or for posterity to read, it matters not which: let the book await its reader for a hundred years; God himself has waited six thousand years for his work to be seen." That is the mood a pale reflection of which the harmonicist is sometimes able to experience when an unexpected discovery is revealed to him in the course of his work.

However this mood also brings with it a danger of which the harmonicist must always be aware: The danger of wanting to see connections of a harmonic variety where they are in reality perhaps only being projected into the structures – in fact from today's viewpoint it cannot be decided with certainly even upon a closer investigation of Kepler's planetary harmonies whether they have the value Kepler assigned them. That is already difficult to decide because there are no unambiguous rules for the establishment of harmonic structures, because these are always found in the polar tension between quantitative exactitude on the one hand and recognizable form on the other. This is a fundamental problem for the harmonic research of today but one into which I cannot go here in greater detail.

In the following period, natural research proceeded to an ever greater extent in the direction of a mechanistic way of thinking of the kind we are accustomed to find in the science of today – but an excessively one-sided view is presented of the motives of the researchers. Hardly anyone today knows, for example, that the physicist Isaac Newton (1643-1727) wrote far more about theology and philosophy than about physics, and the motivation for his work was far more similar to that of Kepler than to that of a modern scientist.

The impulse of Pythagorean thought was sustained more strongly in philosophy – Gottfried Wilhelm Leibniz (1646-1716), for example, wrote that "Music is an unconscious exercise in mathematics in which the mind is unaware that it is calculating" and Arthur Schopenhauer (1788-1860) in his work Die Welt als Wille und Vorstellung (The World as Will and Representation) treats the phenomenal world or nature and music as two different expressions of the

same thing and concludes with the statement that if we succeeded in giving a perfectly accurate and complete explanation of music which went into detail, this would also be a sufficient explanation of the world in concepts, and hence the true philosophy. This led him to reformulate Leibniz's proposition as follows: "Music is an unconscious exercise in metaphysics in which the mind is unaware that it is philosophizing."

Harmonics today

In the nineteenth century, the intellectual heritage of the Pythagoreans was almost entirely forgotten – it was then only of historical interest and regarded by the then triumphant materialist science as not to be taken seriously. During this period, in 1868, a two volume work was published entitled Die harmonikale Symbolik des Altertums (Harmonic Symbolism in Antiquity) by Albert von Thimus (1806-1878), a Cologne jurist and independent scholar, in which he attempted to reconstruct Pythagorean thought and set it in the context of a musical world view of all known ancient cultures. His philological work, which demonstrates his comprehensive mastery of the old sources, gave fresh impetus to harmonics for our time – above all through the discovery of the Lambdoma, a harmonic diagram containing such a rich and deep symbolism in its structure that it can probably never be exhausted. We shall speak again of this later.

Now I have arrived with my examination at the twentieth century and the man whose name is most closely associated with harmonics for our time: Hans Kayser (1891-1964). Kayser initially went through a musical education but soon discovered his bent for philosophy and mysticism; he initiated with the publishing house Insel-Verlag a series entitled Der Dom - Bücher deutscher Mystik (The Cathedral – Books of German Mysticism) and was for many years its principal publisher. This series also included a volume on Johannes Kepler – and a study of Kepler's world of ideas was the decisive stimulus for Kayser's own work. Other influences included the above-mentioned work of Albert v. Thimus and the work of the crys-

tallographer Victor Goldschmidt (1853-1933), who had discovered musical laws in the structure of crystals and using techniques he himself had developed investigated also the spacing of the planets and other relationships in nature.

It was here that Kayser found the field of research that suited him best and from this time onward his entire life was dedicated to harmonic work. His first publication on the subject of harmonics was entitled Orpheus – Vom Klang der Welt (Orpheus – The Sound of the World) and dealt primarily with the harmonics of crystals. This was followed – to mention but a few works – by Der hörende Mensch (The Hearing Human), in which he expanded his research into the harmonic structures to be found in chemistry, astronomy, organic nature, light and colour, and architecture; the Harmonia Plantarum, a development of the morphology of plants out of harmonic basic elements; the short book Akroasis, which was designed as an intelligible-to-all introduction to the intellectual world of harmonics and that is still available today (the word Akroasis = hearing was coined by Kayser as another name for harmonics, which term was often misunderstood) and the comprehensive Lehrbuch der Harmonik (Textbook of Harmonics), in which he presented the results of his investigations in textbook format.

What is being accomplished today in the field of harmonic research can for the most part thank the work of Hans Kayser for its stimuli. Here we should mention above all the Hans-Kayser-Institut für harmonikale Grundlagenforschung (Hans Kayser Institute for Basic Research in Harmonics) at the Hochschule für Musik und darstellende Kunst (University of Music and the Performing Arts) in Vienna, which was founded by Rudolf Haase, a pupil of Hans Kayser. Haase held there what is currently the only chair in harmonics in this sense in the world, now occupied by his successor Werner Schulze. Haase conducted harmonic research in a more scientific-inductive sense than Kayser, whose approach was more deductive and derived from mysticism. Haase, too, wrote many books on harmonics – among them the Geschichte des harmonikalen Pythagoreismus (History of Harmonic Pythagoreanism); Der meßbare

Einklang (The Measurable Unison), in which he presents the connections between harmonics and the results of natural science; the Harmonikale Synthese (The Harmonic Synthesis), which represents an introduction to harmonics; and a biography of Hans Kayser entitled Hans Kayser - Ein Leben für die Harmonik der Welt (Hans Kayser – A Life for the Harmonics of the World).

In Bern, where Kayser spent the second half of his life, there is the Kreis der Freunde um Hans Kayser founded by Walter Ammann and Karl Ledergerber - a circle of friends that also publishes books on harmonics and organizes regular lectures on harmonics.

In Munich, there exists the Arbeitskreis Harmonik, led by the author of this article, which organizes some twelve lectures and courses in harmonics annually at the Freies Musikzentrum München – not only introductions to harmonics but also presentations on individual special subjects. The Arbeitskreis Harmonik is conceived as a forum of exchange for all who work in this area.

PART 2: HARMONIC SYMBOLISM

The Lambdoma – order and symbol

This account of the intellectual history of harmonics and the areas with which it is concerned may perhaps already have conveyed an inkling of its contents. This is not the correct context in which to represent the contents themselves in detail – I would simply like to pick two examples that illustrate the way of thinking of harmonics. The first is a diagram that plays a central role in harmonics and that is at the same time both a mathematical structure and a symbol: the Lambdoma, which we have already briefly mentioned.

The Lambdoma was developed by Albert von Thimus – in his opinion a reconstruction of a diagram of the Pythagoreans, in reality probably an ingenious new construction in the spirit of Pythagoreanism. The Lambdoma is an order diagram of all conceivable interval proportions – or "tone-numbers" as Kayser called them – which in its original form was shaped like the Greek letter lambda.

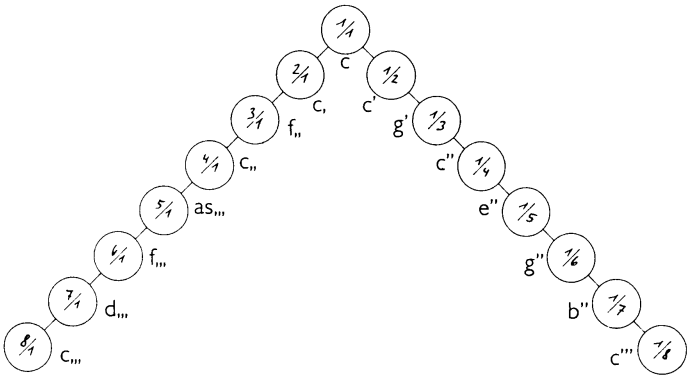


Figure 5: Lambdoma side-rows

As already described, each number proportion is identical with a musical interval that can be heard when the ratios in question are set as string lengths on the monochord. The ratio 1:2 for example yields the octave, 2:3 the fifth, 3:4 the fourth, 4:5 the major third etc.

The side-rows of the Lambdoma represent this succession of intervals starting from any fundamental tone – in this case C – once in ascending, and once in descending, sequence. The ascending sequence is found as a natural overtone series in each sounding tone, such as a vibrating string; the descending sequence is the reflection of the same intervals and is known as the undertone series.

One can now fill in the space between the side-rows by erecting an overtone series above each undertone or vice versa and arrive at the complete representation of the Lambdoma:

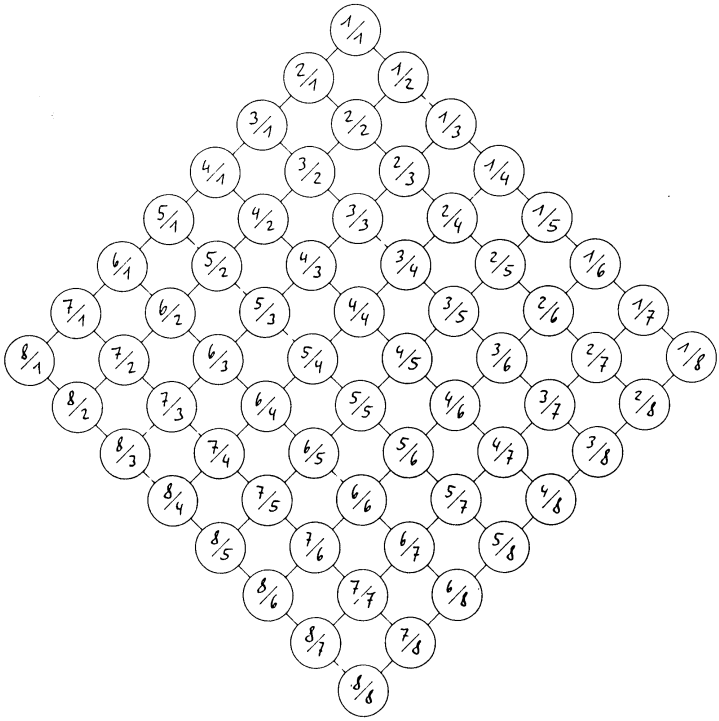


Figure 6: The complete Lambdoma

If one imagines this diagram infinitely extended, one has an order pattern of all conceivable tone-numbers – or, in the symbolic sense, an image of all "Coordinate values of being", of the relationships of all things of the world. If one examines this diagram more closely, it is found to contain a wealth of structures of a mathematical and symbolic nature that one had previously not read into it – this phenomenon is found in all constructions that carry in themselves an 'internal rightness' and therefore imply something more than themselves. I should like to point out just one structure: if you connect all the fractions that are of equal value (i.e. that form the same interval) with a line, they lie on a straight line – known as an equal tone line –

and all equal tone lines intersect at a point outside the diagram, which in a logical extension of the Lambdoma must be given the designation 0/0. The value 0/0 is intellectually and mathematically incomprehensible. Its nature, however, can be guessed at through study of the structure of the Lambdoma, and it can be understand as an image that all things in the world refer to a common origin outside.

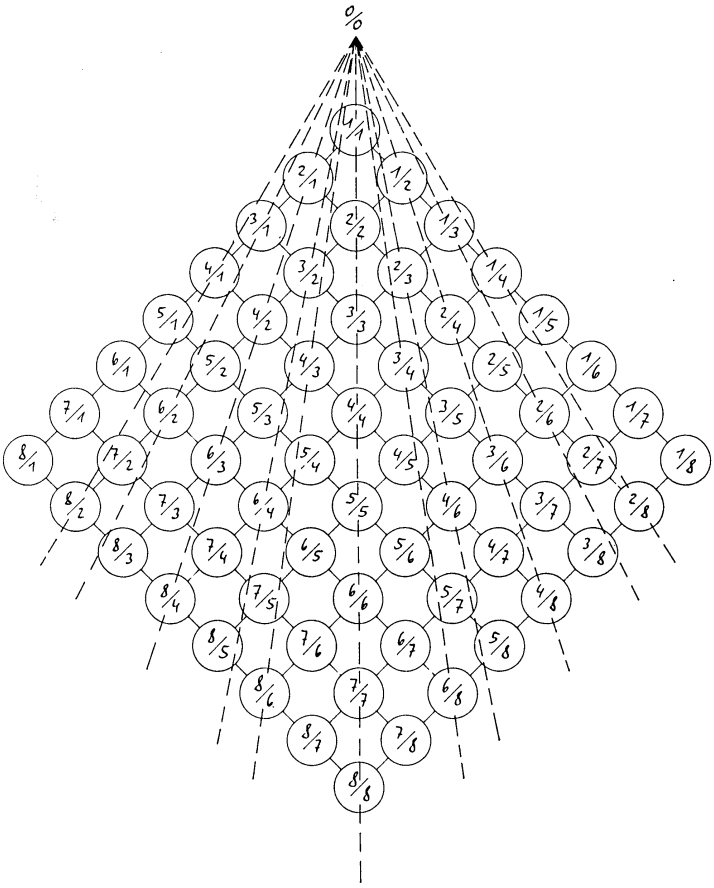


Figure 7: Lambdoma with equal tone lines

The tonal structure of the number world

Whilst with the study of the Lambdoma we were dealing with the structures yielded by a formal ordering of the tone-numbers, I should like in closing to examine the quality of the numbers themselves that becomes accessible if we make them audible as tone-numbers. We will see from this that not only does every individual number have its own quality and form but also that the numbers in their reciprocal natural relationships represent something akin to a symbolic image of the archetypal order of the world.

To this end let us consider each of the individual numbers in turn and listen with a view to determining the role it plays in the tonal organism. It is part of the essence of the thing to do this not only with words but where possible to listen to the tone-numbers and intervals on the monochord or another instrument so as to be able to form a vivid impression of them.

We have, then, as the first number the **One** – which on the monochord is the entire string or the fundamental of the system; for the sake of simplicity, we can call it C. In this way of looking at things, the One is not simply one number among many others but represents the whole, which through the other numbers is dispersed to its different aspects. All the other numbers relate to the One as the tones of a tone system relate to their fundamental tone.

The next number to appear is the **Two** – we have already seen that the Two or halving of the string length produces the octave. It is the most natural and at the same time remarkable phenomenon in music that a note having double the number of vibrations and therefore sounding an octave higher than the starting note is felt to be musically identical to the fundamental and is given the same tone designation – in our example, then, once again C. Whilst the One merely set a reference and identification point in the tonal space, the Two leads to a cyclical structuring of the tonal space in which the following numbers or tones find their places. The further halvings lead to the numbers 4, 8, 16 etc., in other words powers of the Two, which

can be understood as its later manifestations or incarnations in the tonal space.

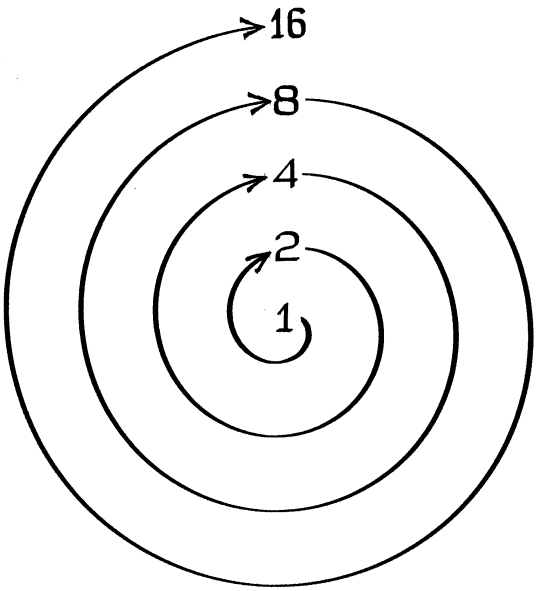


Figure 8: The cyclical structure of the tonal space

The first cycle, which is formed by the One and the Two, is an 'empty' octave – in the following cycle, between the Two and the Four the Three appears as the next number. Viewed structurally, in the continuation the next highest cycle in each case is filled with twice as many numbers, whereby each number that appears in a given cycle also has its tonal representative in the succeeding cycles; in this way, the Three appears in the next octaves as 6, 12, 24 etc. That leads us to this picture of the cyclical ordering of all tone-numbers, which also forms the basis for our further observations. In this representation, the circle corresponds to one octave, and the

same directions in the circle are always the same tones, proceeding outwards in increasing octave intervals.

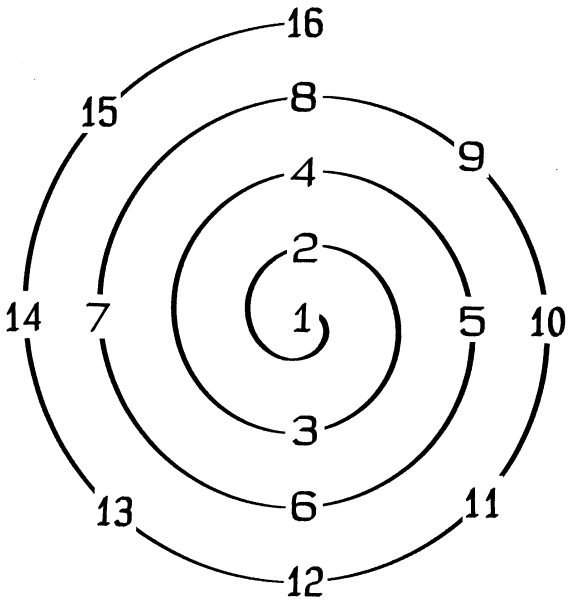


Figure 9: The cyclical structure of the tone-numbers

Whilst the Two had still not produced any really new tone but simply represented the fundamental in higher octaves, with the **Three** appears for the first time a totally new tone: dividing the string length by a third produces the fifth (above the octave), in our example, relative to C that would be a G. From an arithmetic standpoint, the Three is equidistant, obviously, from the Two and the Four – musically, however, it divides the octave into two uneven parts: a fifth and a fourth.

If we listen to the way these intervals present themselves within the octave, the step from the fundamental to the fifth appears as a

being introduced to the octave space whilst the fourth as the step from the fifth tone to the octave tone evokes the impression of a return, a reference by the being placed in the space to its own origin. We could say, then, that the Three polarizes the octave space into these two aspects.

With the appearance of the fifth as the first new tone, immediately the potential exists for an infinite variety of tones: By superimposing ever more fifths through the operation of the Three, we obtain through the circle of fifths a first approximation to the division of the tonal space into twelve parts; since however this circle never completely closes, the tonal space eventually becomes infinitely dense: With the Three, the foundation stone for all other tones is laid – at least, the material for them; it is through the additional numbers, however, that the new tones become independent beings.

The **Four** as the next octave of the Two brings nothing new – it concludes the cycle in which the Three had manifested itself and begins at the same time a new one: the octave from Four to Eight. In this cycle, the **Six** fills the role of the Three that appeared previously – in the space between the Four and the Six appears the Five as a new element and polarizes the space of the fifth in a manner analogous to that in which previously the Three polarized the octave space.

If we listen to the tone that is generated by the **Five**, we find the major third above the fundamental, in our example an E. The fifth is therefore decomposed by the Five to the major third downwards and the minor third upwards - the numbers Four-Five-Six yield the major chord. Whilst, then, in the previous cycle we experienced the empty fifth as a ‘lonely’ and ‘objective’ being placed in the space, this fifth in the next cycle is polarized by the Five – major and minor become possible, the hearing experience gains a ‘human’ element that addresses the feelings.

Analogous to the polarization of the fifth by the Five, the **Seven** appears now in the same cycle between the Six and the Eight and polarises the fourth in a similar manner. In the process, two further

intervals are produced that we are still unable to describe clearly in our musical terminology, because the intervals produced by the Seven have as yet hardly been integrated into our musical experience. Based on the hearing experience, the new tone corresponds to some degree with a minor seventh or in our example a Bb, which allows the chord that was formed in this cycle to become a seventh chord. Whilst, then, in the previous cycle we had experienced the fourth as a reference by the Fifth-being to its own origin, the Seven appears here in this relationship – in terms of hearing experience as posing a question relativizing this relationship. This question can be answered musically in a wide variety of ways.

With the **Eight** begins once again a new cycle, in which the numbers that have already arisen are again represented – the Three or Six through the Twelve, the Five through the Ten, the Seven through the Fourteen. Through the agency of the new numbers, new tones appear once again in the intervals already produced – the Nine for example between the Eight and the Ten, which polarizes the major third into two unequal whole tones that are only found in just intonation – not on today's pianos, which are tuned to equal temperament. Through the whole tones, the element of progression enters the musical world – whereas before the musical space was given tension and structure by the larger intervals, with the **Nine** the idea of constructing a tone scale out of smaller elements is born.

A harmonical cosmogony

In this manner, theoretically all numbers can be examined to discern their specific character and quality – in practice, however, this investigation comes up against limits imposed by our musical experience-capability. Already the numbers Eleven and Thirteen are integrated into our musical experience even less than the Seven and are difficult to grasp through pure listening. Here a careful empathy combined with structural investigations can lead further – for present purposes, however, the image of the numbers in its current state of

elaboration will suffice. One notices also that the smaller the numbers the greater their archetypal potency.

So far we have represented the structure of the number world and its musical character purely descriptively and attempted in the process to grasp its quality through listening. In what follows, I will show that this structure also possesses symbolic characteristics and represents the archetype of a cosmology in its most abstract form. For this, we must first consider what a symbol actually is.

The word 'Symbol' is used in many different senses, and if one considers these more closely, one finds essentially three different levels. Erich Fromm describes these as the "conventional", the "accidental" and the "universal" symbol. These concepts can also be used in our context. It is often the case that one and the same symbol already possesses in itself these three strata. I will show this using as an example the symbol "Cross".

In the outermost stratum, the cross can be a "conventional symbol", such as when it is understood as a mere sign for Christianity, simply the recognition sign upon which people have agreed through convention, in the same way that a meaning is fixed for a traffic sign.

In the next stratum lies the associative 'charging' of the symbol with the totality of all experiences and feelings of men in the course of history. That can be described as 'accidental' in so far as different men in their individual biographies also have had quite different experiences with it and with the appearance of this symbol consequently very different feelings and thoughts are associated.

In the deepest stratum, the meaning of the symbol is no longer dependent upon the individual human, because the symbol - such as a geometric figure – carries an unambiguous meaning within itself, therefore upon more detailed examination "expresses itself". I will attempt to show this using the example of the cross, and through a imaginary geometric investigation.

If we consider a point on a plane that is in the field of tension midway between two other points. It can move either towards the

one point or towards the other, that means, its entire movement possibility is – speaking in terms of content – thematized by the polar tension between the two other points. The only exception is movement perpendicular to a straight line joining the two points: the only direction that leads outwards beyond the theme of the original points and that represents the quintessential Other. Even this new direction is, once again, in itself polar: in this way the cross comes into being. To transform the meaning again from the most abstract level, where it expresses itself, to one that is more easily grasped, one could say from now on that that the cross can be understood as an image for one level of being, that is penetrated by the quintessential and necessary "other".

The world of numbers is in itself also just such a universal symbol that expresses its own meaning. The numbers are the most abstract things of all and in this respect accessible only with difficulty to interpretation, but through tonal structuring and listening we have obtained a basis from which to get closer to their meaning. If I now attempt an interpretation of this structure, we must remain conscious that it is a question here of images that are also culturally determined and that should hint at the actual content. Such an interpretation can take place in the language of mythology, for example, or depth psychology – I choose the image of the Biblical account of the creation, also in order to show that the structure of the world of numbers can be understood as an archetype of all creation myths.

The **One**: Tonal space is in itself unstructured, chaotic, the only reference point is the fundamental – the One – the Spirit of God that comprehends all: "darkness was upon the face of the deep; and the spirit of God moved over the waters."

The **Two**: In the tonal space, the Two produces the first polarization that simultaneously leads to its cyclical octave structure. The first act of creation is a polarization into light and darkness, which at the same time leads to a cyclical phenomenon: Day and Night. "And God said: Let there be light! And there was light... and God sepa-

rated the light from the darkness." Also the further acts of creation are polarizations in the sense of the Two: Heaven and Earth, Water and Land. Other creatures have not yet made their appearance.

The **Three**: In the tonal space, in the second cycle, the Three appears in the shape of the fifth as the first new being. At the same time, with this first being, through the possibility of the circle of fifths, the entire variety of tones in their organic order comes into being. Through this double aspect the Three represents not only the variety of created nature but also its unified representation in man – in the sense of Adam Kadmon, primordial man. In the cycle of the first octave, he is inserted as the first and under this aspect only being as created: the fifth – and the back-reference of the created to the creator: the fourth. The image of man giving the other created beings their names corresponds to that of the creation of the further tones by the fifth.

The **Five**: In the next octave cycle, between Four and Eight what existed previously is further differentiated. In the fifth appears the Five and polarizes it into the major and minor third. Major and minor appear, male and female. Whilst initially the empty fifth was experienced as something objective, superhuman, through the thirds a specifically human element is added. In the first creation cycle, man was an androgynous being – "The Creature" as an image for the totality of creation – at this stage he obtains for the first time through his polarization into man and woman a truly human quality in its narrow sense.

The **Seven**: In the same cycle as that in which the fifth is polarized into the thirds, the Seven enters the fourth. We have experienced the Seven tonally as a question that arises, in that it transforms the pre-existing triad into a seventh chord and therefore calls it into question. We have already experienced the fourth in the preceding cycle as a reference by the created to the creator – in this cycle, now, in which the creature through its polarization has become an actual human, at the same time in its back-reference to the creator a ques-

tion arises: the question of knowledge and the promise of the snake, to be like God.

The **Nine**: With the Eight another cycle is closed; with the following Nine begins the progressive, the construction of the tone scale out of smaller elements. So the paradisal space must be abandoned after the Eight and earthly life begins, and with it the need to get bread “by the sweat of your brow”.

To this way of looking at things one could add many details that would show the parallels of the two levels – but I would certainly not wish to convey the impression that a ‘system of interpretation’ were already in place – these interpretations should be understood as nothing more than an attempt to deal with symbolic contents in such a way that one penetrates them on their absolute and objective level, that one can portray them on a human level as images. The images themselves however are interchangeable and serve only to illustrate the abstract meaning of the archetypical symbol.